Function Terminology and Notation

If you don't know how to read, you won't get very far in school. Well, math has its own words and notation – it's almost an entirely new language (want to impress your math professor? Start asking them about "homeomorphisms," "isometries," or "monoids"...there are even things with sillier names, like "perverse sheaves" and "unknots"). You have to know the words and notation, or we can't communicate about math, and if we can't communicate about it, then it's impossible to learn. Since functions are ubiquitous in mathematics, you'll want to have function terminology and notation down pat.

When we deal with functions, we typically (but not always!) start by defining a *formula* for the function. Remember what a function is? It's a rule that assigns something in the function's range to each thing in the function's domain. Or, intuitively, it's a machine that takes inputs, processes them somehow, and spits out outputs. Often we say what that machine does by specifying a formula. We'll write something like:

$$f(x) = 7x^2 - 3.$$

This notation is telling us several things:

- The name of the function is f. It's f for Function. We could have called the function Bob, but f is probably a better name for a function (no offense to any Bobs in the class).
- The variable we're using to represent inputs into the function is x. So any other variable you see around will represent a constant (more on that later). The way we'll describe this is by saying "f is a function of x."
- What this function does is take in a number, square it, then multiply it by 7, and then subtract 3 from that and spit out the result.

Now that we've specified what the function does using this formula, we can start plugging things into it. We could plug in actual numbers:

$$f(3) = 7 \cdot 3^2 - 3 = 7 \cdot 9 - 3 = 63 - 3 = 60.$$

But we don't have to be limited to that. Sometimes we want to plug in entire expressions:

$$f(1+t) = 7(1+t)^2 - 3 = 7(1 + 2t + t^2) - 3 = 7 + 14t + 7t^2 - 3 = 4 + 14t + 7t^2.$$

Sometimes those expressions will involve x. This isn't very good notation, but it's the way it is and we're stuck with it.

$$f(x^2) = 7(x^2)^2 - 3 = 7x^4 - 3.$$

Sometimes we'll plug in other functions!

$$f(g(x)) = 7(g(x))^2 - 3.$$

Hopefully, you're getting the idea. Once we've defined the function via a formula, we know how to plug in anything else to the function.

$$f(\text{cookie monster}) = 7 \cdot (\text{cookie monster})^2 - 3.$$

Can you answer these questions?

(1) Let $f(x) = \frac{1}{x} + x^2$.

(a) What is $f(4)$?

(b) What is $f(-1)$?

(c) What is $f(\frac{1}{x})$? (Simplify as much as possible.)

(2) Let $R(t) = 4t - t^2$. What is $R(x-5)$? Simplify your answer.

(3) Let $Q(z) = x + 2z$.

(a) What is Q a function of?

(b) What is $Q(3)$?

(c) What is $Q(a)$?

(4) If f is a function, then which is impossible?

(a) $f(2) = 4$ and $f(8) = 4$.
(b) $f(4) = 2$ and $f(4) = 8$.

Notice we haven't seen any y's around? But there's always y's around with functions, right? That's because of the very common equation:

$$y = f(x).$$

Now, we don't actually need y's to do anything. By the notation we've defined so far, x represents an arbitrary input and $f(x)$ denotes the corresponding output. (Go look back at our first example. If we input 3 into the function defined by $f(x) = 7x^2 - 3$, then our output is 60. In other words, $f(3) = 60$. So 3 is the input, and $f(3)$ is the output.) But, it's kind of a pain to write $f(x)$ all the time, so some smart person a long time ago decided to write y instead. Hence, we often write $f(x) = y$, and $f(x)$ and y both represent the output of a function.

(This is actually a confusing point. The notation $f(x)$ can really mean a few things: it can represent the formula for a function (e.g., $f(x) = x^2$) or it can represent the output of a function (e.g., $f(x) = y$ or $f(x) = 3$). Also, students (and sometimes professors) will write $f(x)$ to denote the function itself, f. So you may have to rely on context to figure out what is meant.)

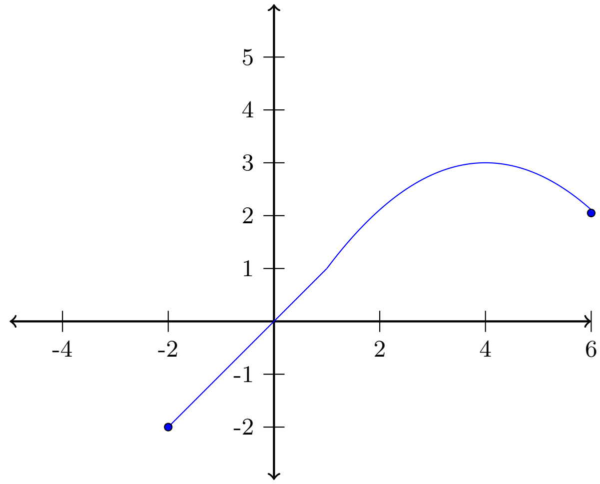
When you talk about the "y-axis" on graphs, what you're really referring to is the "output" axis, the place where we record outputs. High up on the axis means a big output, low down on the axis means a small output, and so on.

(5) If the point $(3, 5)$ is on the graph of a function, what does that mean?

(6) If the value 7 is in the domain of a function, what does that mean in terms of its graph?

(7) If the value 7 is in the range of a function, what does that mean in terms of its graph?

(8) Use the graph of f below to answer the following questions.



(a) What is $f(3)$? (approximately)

(b) What is the domain of the function?

(c) What is the range of the function?